

Contents*

Preface	ix
About the Author	xi
Chapter 6. Differentiation on E^n and Other Normed Linear Spaces	1
1. Directional and Partial Derivatives	1
Problems on Directional and Partial Derivatives	6
2. Linear Maps and Functionals. Matrices	7
Problems on Linear Maps and Matrices	14
3. Differentiable Functions	16
Problems on Differentiable Functions	25
4. The Chain Rule. The Cauchy Invariant Rule	28
Further Problems on Differentiable Functions	33
5. Repeated Differentiation. Taylor's Theorem	35
Problems on Repeated Differentiation and Taylor Expansions	44
6. Determinants. Jacobians. Bijective Linear Operators	47
Problems on Bijective Linear Maps and Jacobians	55
7. Inverse and Implicit Functions. Open and Closed Maps	57
Problems on Inverse and Implicit Functions, Open and Closed Maps	67
8. Baire Categories. More on Linear Maps	70
Problems on Baire Categories and Linear Maps	76
9. Local Extrema. Maxima and Minima	79
Problems on Maxima and Minima	84
10. More on Implicit Differentiation. Conditional Extrema	87
Further Problems on Maxima and Minima	94
Chapter 7. Volume and Measure	97
1. More on Intervals in E^n . Semirings of Sets	97
Problems on Intervals and Semirings	104
2. \mathcal{C}_σ -Sets. Countable Additivity. Permutable Series	104
Problems on \mathcal{C}_σ -Sets, σ -Additivity, and Permutable Series	112

* "Starred" sections may be omitted by beginners.

3. More on Set Families	115
Problems on Set Families	121
4. Set Functions. Additivity. Continuity	124
Problems on Set Functions	133
5. Nonnegative Set functions. Premeasures. Outer Measures	136
Problems on Premeasures and Related Topics	141
6. Measure Spaces. More on Outer Measures	147
Problems on Measures and Outer Measures	155
7. Topologies. Borel Sets. Borel Measures	160
Problems on Topologies, Borel Sets, and Regular Measures	165
8. Lebesgue Measure	168
Problems on Lebesgue Measure	172
9. Lebesgue–Stieltjes Measures	176
Problems on Lebesgue–Stieltjes Measures	178
10. Vitali Coverings	180
Problems on Vitali Coverings	190
11. Generalized Measures. Absolute Continuity	194
Problems on Generalized Measures	206
12. Differentiation of Set Functions	210
Problems on Differentiation of Set Functions	215

Chapter 8. Measurable Functions. Integration **217**

1. Elementary and Measurable Functions	217
Problems on Measurable and Elementary Functions in (S, \mathcal{M})	222
2. Measurability of Extended-Real Functions	224
Further Problems on Measurable Functions in (S, \mathcal{M})	229
3. Measurable Functions in (S, \mathcal{M}, m)	231
Problems on Measurable Functions in (S, \mathcal{M}, m)	238
4. Integration of Elementary Functions	241
Problems on Integration of Elementary Functions	248
5. Integration of Extended-Real Functions	251
Problems on Integration of Extended-Real Functions	265
6. Integrable Functions. Convergence Theorems	267
Problems on Integrability and Convergence Theorems	277
7. Integration of Complex and Vector-Valued Functions	283
Problems on Integration of Complex and Vector-Valued Functions	291
8. Product Measures. Iterated Integrals	293
Problems on Product Measures and Fubini Theorems	302

9. Riemann Integration. Stieltjes Integrals	306
Problems on Riemann and Stieltjes Integrals	319
10. Integration in Generalized Measure Spaces.....	323
Problems on Generalized Integration	333
11. The Radon–Nikodym Theorem. Lebesgue Decomposition	336
Problems on Radon–Nikodym Derivatives and Lebesgue Decomposition	344
12. Integration and Differentiation	345
Problems on Differentiation and Related Topics	354
Chapter 9. Calculus Using Lebesgue Theory	357
1. L-Integrals and Antiderivatives	357
Problems on L-Integrals and Antiderivatives.....	367
2. More on L-Integrals and Absolute Continuity.....	372
Problems on L-Integrals and Absolute Continuity	384
3. Improper (Cauchy) Integrals	387
Problems on Cauchy Integrals.....	397
4. Convergence of Parametrized Integrals and Functions	402
Problems on Uniform Convergence of Functions and C-Integrals ..	412
Index	417